

EXPONENTIAL TYPE RATIO AND PRODUCT ESTIMATOR FOR RATIO OF TWO POPULATION MEANS

SHAILENDRA RAWAL¹ & RAJESH TAILOR²

¹Maharaja College, Vikram University, Ujjain, Madhya Pradesh, India

²School of Studies in Statistics, Vikram University, Ujjain, Madhya Pradesh, India

ABSTRACT

In this paper suggested ratio and product type exponential estimator for ratio of two population mean. Bias and mean squared error of suggested estimator have been obtained suggested estimator have been compared with usual estimator, ratio estimator given by Singh (1965). An empirical study has been carried out to demonstrate the performance of suggested estimator.

KEYWORDS: Ratio and Product Type Estimator, Bias, Mean Square Error

1. INTRODUCTION

This paper deals with the problem of estimation of ratio of two population means. Bahl and Tuteja (1991) suggested exponential type estimator of ratio of two population means. In this p we have proposed modified exponential type estimator for ratio of two population means. Suggested estimators have been compared with usual estimator and ratio type estimator. It has been shown that proposed estimators are more efficient than other considered estimators under certain given conditions.

Let $U = \{U_1, U_2, \dots, U_N\}$ be a finite population of size N and y_0 and y_1 are two study variates. Let x , is a auxiliary variate taking values x_i ($i = 1, 2, \dots, N$). A sample of size n is drawn from N with simple random sampling without replacement an estimator of ratio of two population means.

$$R = \frac{\bar{Y}_0}{\bar{Y}_1}$$

to estimate ratio of two population mean R usual estimator is

$$\hat{R} = \frac{y_0}{y_1} \tag{1.1}$$

Let

$$\bar{y}_0 = \bar{Y}_0(1 + e_0), \bar{y}_1 = \bar{Y}_1(1 + e_1) \quad \bar{x} = \bar{X}(1 + e_2)$$

Such that

$$E(e_0) = E(e_1) = E(e_2) = 0, E(e_0^2) = \theta C_0^2, E(e_1^2) = \theta C_1^2, E(e_2^2) = \theta C_2^2$$

$$E(e_0e_1) = \theta\rho_{01}C_0C_1, E(e_0e_2) = \theta\rho_{02}C_0C_2, E(e_1e_2) = \theta\rho_{12}C_1C_2$$

Now we express R in terms of e_i s we have

$$\hat{R} = \frac{y_0}{y_1}$$

$$\hat{R} - R = R(e_0 - e_1 + e_1^2 - e_0e_1) \quad (1.2)$$

Taking expectation of both sides of (1.2) we get

$$\text{Bias}(\hat{R}) = R\theta(C_1^2 - \rho_{01}C_0C_1) \quad (1.3)$$

Taking square and expectation of both sides of (1.3), upto the first degree of approximation mean squared error of \hat{R} is

$$MSE(\hat{R}) = R^2\theta(C_0^2 + C_1^2 - 2\rho_{01}C_0C_1) \quad (1.4)$$

where

$$S_0^2 = \frac{1}{N-1} \sum_{i=1}^N (y_{0i} - \bar{Y}_0)^2, S_1^2 = \frac{1}{N-1} \sum_{i=1}^N (y_{1i} - \bar{Y}_1)^2, S_2^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{X})^2$$

$$C_0^2 = \frac{S_0^2}{\bar{Y}_0^2}, C_1^2 = \frac{S_1^2}{\bar{Y}_1^2}, C_2^2 = \frac{S_2^2}{\bar{X}^2}$$

$$S_{01} = \frac{1}{N-1} \sum_{i=1}^N (y_{0i} - \bar{Y}_0)(y_{1i} - \bar{Y}_1), S_{02} = \frac{1}{N-1} \sum_{i=1}^N (y_{0i} - \bar{Y}_0)(x_i - \bar{X}),$$

$$S_{12} = \frac{1}{N-1} \sum_{i=1}^N (y_{1i} - \bar{Y}_1)(x_i - \bar{X})$$

$$\rho_{01} = \frac{S_{01}}{S_0S_1}, \rho_{02} = \frac{S_{02}}{S_0S_2}, \rho_{12} = \frac{S_{12}}{S_1S_2}$$

$$\theta = \left(\frac{1}{n} - \frac{1}{N} \right).$$

Using information of an auxiliary variate x, ratio estimator of ratio of two population mean is defined as

$$\hat{R}_R = \hat{R} \left(\frac{\bar{X}}{\bar{X}} \right) \quad (1.5)$$

$$\hat{R}_R - R = R(e_0 - e_1 - e_2 + e_1^2 + e_2^2 + e_1e_2 - e_0e_1) \quad (1.6)$$

Taking expectation of both sides of (1.6) we get

$$\text{Bias}(\hat{R}_R) = R\theta(C_1^2 + C_2^2 + \rho_{12}C_1C_2 - \rho_{01}C_0C_1) \tag{1.7}$$

Taking square and expectation of both sides of (1.7), upto the first degree of approximation is mean squared error of \hat{R}_R is

$$\text{MSE}(\hat{R}_R) = R^2\theta(C_0^2 + C_1^2 + C_2^2 - 2\rho_{01}C_0C_1 - 2\rho_{02}C_0C_2 + 2\rho_{12}C_1C_2) \tag{1.8}$$

where $C_2 = \frac{S_x}{\bar{X}}$

C_0 = coefficient of variation y_0

C_1 = coefficient of variation y_1

C_2 = coefficient of variation x

Bahl and Tuteja (1991) suggested exponential type ratio and product type estimator for population mean as

$$\hat{Y}_{Re} = \bar{y} \exp\left[\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right] \tag{1.9}$$

$$\hat{Y}_{Pe} = \bar{y} \exp\left[\frac{\bar{X}_2 - \bar{x}_2}{\bar{X}_2 + \bar{x}_2}\right] \tag{1.10}$$

2.2 SUGGESTED ESTIMATOR

In the line of Bahl and Tuteja (1991), proposed ratio and product type estimator for ratio of two population means are

$$\hat{R}_{Re}^R = \hat{R} \exp\left[\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right] \tag{2.1}$$

$$\hat{Y}_{Pe}^R = \bar{y} \exp\left[\frac{\bar{X}_2 - \bar{x}_2}{\bar{X}_2 + \bar{x}_2}\right] \tag{2.2}$$

When x_1 and x_2 are positively and negatively correlated auxiliary variate with (y_0, y_1) respectively.

$$\hat{R}_{Re}^R - R = R\left[e_0 - e_1 - \frac{e_2}{2} + e_1^2 + \frac{3e_2^2}{8} - e_0e_1 + \frac{e_1e_2}{2} - \frac{e_0e_2}{2}\right] \tag{2.3}$$

Taking expectation of both sides of (1.6) we get

$$\text{Bias}(\hat{R}_{Re}^R) = R\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_1^2 + \frac{3C_2^2}{8} - \rho_{01}C_0C_1 + \frac{\rho_{12}C_1C_2}{2} - \frac{\rho_{02}C_0C_2}{2}\right] \tag{2.4}$$

Taking square and expectation of both sides of (1.7), upto the first degree of approximation is mean squared error

of \hat{R}_{Re}^R is

$$MSE(\hat{R}_{Re}^R) = R^2 \theta \left(C_0^2 + C_1^2 + \frac{C_2^2}{4} - 2\rho_{01}C_0C_1 - \rho_{02}C_0C_2 + \rho_{12}C_1C_2 \right) \quad (2.5)$$

Similarly bias and mean squared error of suggested estimator is obtained are

$$Bias(\hat{R}_{Pe}^R) = R\theta \left[C_1^2 + \frac{3C_3^2}{8} - \rho_{01}C_0C_1 + \frac{\rho_{12}C_1C_3}{2} - \frac{\rho_{02}C_0C_3}{2} \right] \quad (2.6)$$

$$MSE(\hat{R}_{Pe}^R) = R^2 \theta \left(C_0^2 + C_1^2 + \frac{C_2^2}{4} - 2\rho_{01}C_0C_1 - \rho_{02}C_0C_2 + \rho_{12}C_1C_2 \right) \quad (2.7)$$

2.3 BIAS COMPARISION

Some times bias of an estimator is considered as disadvantageous. So in this section we compare the bias of proposed estimators with the biases of other considered estimators.

Bias of usual estimator \hat{R} and ratio of estimator \hat{R}_R are

$$Bias(\hat{R}) = R\theta(C_1^2 - \rho_{01}C_0C_1) \quad (3.1)$$

$$Bias(\hat{R}_R) = R\theta(C_1^2 + C_2^2 + \rho_{12}C_1C_2 - \rho_{01}C_0C_1) \quad (3.2)$$

Comparing (2.4) and (3.1) it is observed that the bias of the proposed estimator \hat{R}_{Re}^R is less then the bias of usual estimator \hat{R} ,

i.e.

$$\begin{aligned} |B(\hat{R}_{Re}^R)| &< |B(\hat{R})| \\ \Rightarrow \left(\frac{3C_2^2}{8} + \frac{\rho_{12}C_1C_2}{2} - \frac{\rho_{02}C_0C_2}{2} \right) \\ &\quad \left(2C_1^2 + \frac{3C_2^2}{8} - 2\rho_{01}C_0C_1 + \frac{\rho_{12}C_1C_2}{2} - \frac{\rho_{02}C_0C_2}{2} \right) < 0 \end{aligned}$$

Comparing (2.4) and (3.1) it is observed that bias of the proposed estimator \hat{R}_{Re}^R is less then \hat{R}_R

i.e.

$$|B(\hat{R}_{Re}^R)| < |B(\hat{R}_R)|$$

$$\Rightarrow \left(\frac{5}{8}C_2^2 - \frac{\rho_{02}C_0C_2}{2} - \frac{\rho_{12}C_1C_2}{2} \right) \left(2C_1^2 + \frac{11}{8}C_2^2 - 2\rho_{01}C_0C_1 + \frac{3}{2}\rho_{12}C_1C_2 - \frac{\rho_{02}C_0C_2}{2} \right) < 0$$

- Similarly condition under the bias estimator of \hat{R}_{Re}^R is less than \hat{R} is

$$\begin{aligned} & |B(\hat{R}_{Re}^R)| < |B(\hat{R})| \\ \Rightarrow & \left(\frac{3C_3^2}{8} + \frac{\rho_{13}C_1C_3}{2} - \frac{\rho_{03}C_0C_3}{2} \right) \left(2C_1^2 + \frac{3C_3^2}{8} - 2\rho_{01}C_0C_1 + \frac{\rho_{13}C_1C_3}{2} - \frac{\rho_{03}C_0C_3}{2} \right) < 0 \end{aligned}$$

Comparing (2.6) and (3.1) it is observed that bias of the proposed estimator \hat{R}_{Re}^R is less than \hat{R}_R

i.e.

$$\begin{aligned} & |B(\hat{R}_{Re}^R)| < |B(\hat{R}_R)| \\ \Rightarrow & \left(\frac{5}{8}C_3^2 - \frac{\rho_{03}C_0C_3}{2} - \frac{\rho_{13}C_1C_3}{2} \right) \left(2C_1^2 + \frac{11}{8}C_3^2 - 2\rho_{01}C_0C_1 + \frac{3}{2}\rho_{13}C_1C_3 - \frac{\rho_{03}C_0C_3}{2} \right) < 0 \end{aligned}$$

2.4 EFFICIENCY COMPARISON

Mean squared error of usual estimator \hat{R} an ratio type estimator \hat{R}_R

$$MSE(\hat{R}) = R^2\theta(C_0^2 + C_1^2 - 2\rho_{01}C_0C_1) \tag{4.1}$$

$$MSE(\hat{R}_R) = R^2\theta(C_0^2 + C_1^2 + C_2^2 - 2\rho_{01}C_0C_1 - 2\rho_{02}C_0C_2 + 2\rho_{12}C_1C_2) \tag{4.2}$$

- Comparison of (2.5) and (4.1) shows that suggested estimator \hat{R}_{Re}^R is more efficient than usual estimator \hat{R} if

$$\begin{aligned} & MSE(\hat{R}_{Re}^R) < MSE(\hat{R}) \\ & R^2\theta \left(C_0^2 + C_1^2 + \frac{C_2^2}{4} - 2\rho_{01}C_0C_1 - \rho_{02}C_0C_2 + \rho_{12}C_1C_2 \right) < R^2\theta(C_0^2 + C_1^2 - 2\rho_{01}C_0C_1) \\ & C_2 \left(\frac{C_2}{4} - \rho_{02}C_0 + \rho_{12}C_1 \right) < 0 \end{aligned}$$

$$C_2 < 0 \text{ and } \frac{C_2}{4} - \rho_{02}C_0 + \rho_{12}C_1 > 0$$

$$\text{either } C_2 < 0 \text{ and } C_2 > 4(\rho_{02}C_0 - \rho_{12}C_1)$$

$$\text{or } C_2 > 0 \text{ and } \frac{C_2}{4} - \rho_{02}C_0 + \rho_{12}C_1 < 0$$

$$\Rightarrow C_2 > 0 \text{ and } C_2 < 4(\rho_{02}C_0 - \rho_{12}C_1)$$

Thus conditions under which proposed estimator \hat{R}_{Re}^R would be more efficient then \hat{R} is

$$\text{either } 0 < C_2 < 4(\rho_{02}C_0 - \rho_{12}C_1) \text{ or } 4(\rho_{02}C_0 - \rho_{12}C_1) < C_2 < 0$$

- Comparison of (2.5) and (4.2) shows that suggested estimator \hat{R}_{Re}^R is more efficient than usual estimator \hat{R} if

$$MSE(\hat{Y}_{Re}^R) < MSE(\hat{R}_R)$$

$$R^2\theta \left(C_0^2 + C_1^2 + \frac{C_2^2}{4} - 2\rho_{01}C_0C_1 - \rho_{02}C_0C_2 + \rho_{12}C_1C_2 \right) < \\ R^2\theta (C_0^2 + C_1^2 + C_2^2 - 2\rho_{01}C_0C_1 - 2\rho_{02}C_0C_2 + 2\rho_{12}C_1C_2)$$

$$C_2 \left(\frac{-3}{4}C_2 + \rho_{02}C_0 - \rho_{12}C_1 \right) < 0$$

$$C_2 < 0 \text{ and } \frac{-3}{4}C_2 + \rho_{02}C_0 - \rho_{12}C_1 > 0$$

$$\text{either } C_2 < 0 \text{ and } C_2 > \frac{4}{3}(\rho_{02}C_0 - \rho_{12}C_1)$$

$$\text{or } C_2 > 0 \text{ and } \frac{C_2}{4} - \rho_{02}C_0 + \rho_{12}C_1 < 0$$

$$\Rightarrow C_2 > 0 \text{ and } C_2 < \frac{4}{3}(\rho_{02}C_0 - \rho_{12}C_1)$$

Thus conditions under which proposed estimator \hat{R}_{Re}^R would be more efficient then \hat{R}_R is either

$$0 < C_2 < 4(\rho_{02}C_0 - \rho_{12}C_1)$$

$$\text{or } \frac{4}{3}(\rho_{02}C_0 - \rho_{12}C_1) < C_2 < 0$$

- Comparison of (2.7) and (4.1) shows that suggested estimator \hat{R}_{Pe}^R is more efficient than usual estimator \hat{R} if

$$MSE(\hat{R}_{Pe}^R) < MSE(\hat{R})$$

$$R^2\theta\left(C_0^2 + C_1^2 + \frac{C_2^2}{4} - 2\rho_{01}C_0C_1 - \rho_{02}C_0C_2 + \rho_{12}C_1C_2\right) < R^2\theta(C_0^2 + C_1^2 - 2\rho_{01}C_0C_1)$$

$$C_2\left(\frac{C_2}{4} - \rho_{02}C_0 + \rho_{12}C_1\right) < 0$$

$$C_2 < 0 \text{ and } \frac{C_2}{4} - \rho_{02}C_0 + \rho_{12}C_1 > 0$$

$$\text{either } C_2 < 0 \text{ and } C_2 > 4(\rho_{02}C_0 - \rho_{12}C_1)$$

$$\text{or } C_2 > 0 \text{ and } \frac{C_2}{4} - \rho_{02}C_0 + \rho_{12}C_1 < 0$$

$$\Rightarrow C_2 > 0 \text{ and } C_2 < 4(\rho_{02}C_0 - \rho_{12}C_1)$$

Thus conditions under which proposed estimator \hat{R}_{Re}^R would be more efficient than \hat{R} is either

$$0 < C_2 < 4(\rho_{02}C_0 - \rho_{12}C_1)$$

$$\text{or } 4(\rho_{02}C_0 - \rho_{12}C_1) < C_2 < 0$$

- Comparison of (2.7) and (4.2) shows that suggested estimator \hat{R}_{Pe}^R is more efficient than usual estimator \hat{R}_R if

$$MSE(\hat{R}_{Pe}^R) < MSE(\hat{R}_R)$$

$$R^2\theta\left(C_0^2 + C_1^2 + \frac{C_3^2}{4} - 2\rho_{01}C_0C_1 - \rho_{03}C_0C_3 + \rho_{13}C_1C_3\right) < 0$$

$$R^2\theta(C_0^2 + C_1^2 + C_3^2 - 2\rho_{01}C_0C_1 - 2\rho_{03}C_0C_3 + 2\rho_{13}C_1C_3)$$

$$C_3\left(\frac{-3}{4}C_3 + \rho_{03}C_0 - \rho_{13}C_1\right) < 0$$

$$C_3 < 0 \text{ and } \frac{-3}{4}C_3 + \rho_{03}C_0 - \rho_{13}C_1 > 0$$

$$\text{either } C_3 < 0 \text{ and } C_3 > \frac{4}{3}(\rho_{03}C_0 - \rho_{13}C_1)$$

$$\text{or } C_3 > 0 \text{ and } \frac{C_3}{4} - \rho_{03}C_0 + \rho_{13}C_1 < 0$$

$$\Rightarrow C_3 > 0 \text{ and } C_3 < \frac{4}{3}(\rho_{03}C_0 - \rho_{12}C_1)$$

Thus conditions under which proposed estimator \hat{R}_{Pe}^R would be more efficient than \hat{R} is either

$$0 < C_3 < \frac{4}{3}(\rho_{03}C_0 - \rho_{12}C_1)$$

$$\text{or } \frac{4}{3}(\rho_{03}C_0 - \rho_{12}C_1) < C_3 < 0$$

Thus the condition under which suggested estimator \hat{R}_{Re}^R would be more efficient than ratio estimator \hat{R}_R given by Singh (1965) if

$$\text{either } 0 < C_2 < 4(\rho_{02}C_0 - \rho_{12}C_1)$$

$$\text{or } \frac{4}{3}(\rho_{02}C_0 - \rho_{12}C_1) < C_2 < 0$$

Similarly the condition under which suggested estimator \hat{R}_{Pe}^R would be more efficient than ratio estimator \hat{R}_R given by Singh (1965) if

$$\text{either } 0 < C_2 < 4(\rho_{02}C_0 - \rho_{12}C_1)$$

$$\text{or } \frac{4}{3}(\rho_{02}C_0 - \rho_{12}C_1) < C_2 < 0$$

2.5 EMPIRICAL STUDY

To show the performance of the suggested estimator \hat{R}_{Re}^R and \hat{R}_{Pe}^R we are considering two natural data sets. Descriptions of the problem is given below

Population I

y_0 : Wing length

y_1 : Fourth palp length

x : Third palp length

Population I [Source: Johnson & Wichern (2003)]

Table 1

N=10 n=3	$\theta=0.23333$	$R=1.328767$	$R^2=1.765622$
	$\bar{Y}_0=102$	$\bar{Y}_1=35.6$	$\bar{X}=26.5$
	$S_{01}=-0.55556$	$S_{02}=12.66667$	$S_{12}=-0.33333$
	$C_0=0.055073$	$C_1=0.040164$	$C_2=0.127348784$
	$\rho_{01}=-0.11513$	$\rho_{02}=0.668165$	$\rho_{12}=-0.06908$
	$C_0^2=0.003033$	$C_1^2=0.001613$	$C_2^2=0.016218$
	$S_0^2=31.55556$	$S_1^2=2.044444$	$S_2^2=11.38889$
	$S_0=5.617433$	$S_1=1.429841$	$S_2=3.374743$

Population II [Source: Johnson & Wichern (2003)]

(a) For \hat{R}_{Re}^R

\bar{Y}_0 : Male length

\bar{Y}_1 : Male width

\bar{X} : Male height

Table 2

N=24 N=4	$\theta=0.208333$	$R=1.284096$	$R^2=1.648903$
	$\bar{Y}_0=113.375$	$\bar{Y}_1=74$	$\bar{X}=37$
	$S_{01}=79.14674$	$S_{02}=37.375$	$S_{12}=21.65399$
	$C_0=0.103902$	$C_1=0.080121$	$C_2=0.082427$
	$\rho_{01}=0.949785$	$\rho_{02}=0.945558$	$\rho_{12}=0.912265$
	$C_0^2=0.010796$	$C_1^2=0.006419$	$C_2^2=0.006794$
	$S_0^2=138.7663$	$S_1^2=50.04166$	$S_2^2=11.25906$
	$S_0=11.77991$	$S_1=7.074013$	$S_2=3.355452$

(b) For \hat{R}_{Pe}^R

\bar{Y}_0 : Male length

\bar{Y}_1 : Male width

\bar{X} : Male height

Table 3

N=24 N=4	$\theta=0.208333$	$R=1.284096$	$R^2=1.648903$
	$\bar{Y}_0=113.375$	$\bar{Y}_1=37$	$\bar{X}=74$
	$S_{01}=79.14674$	$S_{03}=21.65399$	$S_{13}=37.375$
	$C_0=0.103902$	$C_1=0.082427$	$C_3=0.080121$
	$\rho_{01}=0.949785$	$\rho_{03}=0.912265$	$\rho_{13}=0.945558$
	$C_0^2=0.010796$	$C_1^2=0.006794$	$C_3^2=0.006419$
	$S_0^2=138.7663$	$S_1^2=11.25906$	$S_2^2=50.04166$
	$S_0=11.77991$	$S_1=3.355452$	$S_2=7.074013$

Percent Relative Efficiencies

Table 4

Estimator	Population I	Population II
\hat{R}	100.00	100.00
\hat{R}_R	45.64718	166.0676
\hat{R}_{Re}^R	219.0716	654.3909
\hat{R}_{Pe}^R	107.1302	601.5625

Table 1 shows that suggested estimators \hat{R}_{Re}^R and \hat{R}_{Pe}^R have higher percentage relative efficiency in comparison to \hat{R} and \hat{R}_R . Thus suggested estimator recommended for use in practice.

Section 2.4 provides the conditions under which suggested exponential type estimator are more efficient than usual estimator \hat{R} and ratio estimator \hat{R}_R for ratio of two population means.

7. CONCLUSIONS

Estimation is a common problem in various field if agriculture, economics, population etc. where some parameters like population total, population mean population variance ratio of two population means etc need to be estimates.

In this article we have considered the problem of estimating the population mean of the study variable when the population mean of an auxiliary variable is known in simple random sampling without replacement (SRSWOR). The class of estimators has been proposed and the bias and mean square error expressions of the proposed class of estimators have been obtained up to first degree of approximation.

REFERENCES

1. **Bahl, S. and Tuteja, R.K. (1991).** *Ratio and product type exponential estimator. Introduction and optimization science.* 12, (1), 159-163.
2. **Cochran, W.G. (1940).** *The estimation of the yields of cereal experiments by sampling for the ratio gain to total produce. Jour. Agri. Soc.,* 30, 262-275

3. **Murthy, M.N. (1964).** Product method of estimation. *Sankhya*, 26,A, 284-307.
4. **Pandey, B.N. and Dube, V. (1988).** Modified product estimator using coefficient of variation of auxiliary variable. *Assam Stat. Rev.*, 2, 64-66.
5. **Robson, D.S. (1957).** Application of multivariate polykays to the theory of unbiased ratio-type estimators. *Jour. Amer. Statist. Assoc.*, 52, 511-522.
6. **Searls, D.T. (1964).** The utilization of known coefficient of variation in the estimation procedure. *Jour. Amer. Stati. Assoc.*, 59, 1225-1226.
7. **Singh, G.N. and Rani, R. (2005, 2006).** Some linear transformations on auxiliary variable for estimating the ratio of two population means in sample surveys. *Model assisted statistics and applications*, 1, 3-7.
8. **Singh, H. P., Singh, R., Ruiz Espejo, M. and Pineda M. D. (2005).** On the efficiency of a dual to ratio-cum-product estimator in sample surveys. *Mathematical Proceedings of the Royal Irish Academy*, 105 A (2), 51-56.
9. **Singh, H.P. and Tailor, R. (2003).** Use of known correlation coefficient in estimating the finite population mean. *Stat. Transin.* 6,4, 555-560
10. **Singh, H. P. Tailor, R. Tailor, R. and Kakran, M.S. (2004).** An improved estimator of population mean using power transformation, *J. Indian Soc. Agric. Statist.*, 58, 2, 223-230.
11. **Singh, M.P. (1965).** On the estimation of ratio and product of the population parameters. *Sankhya*, B, 27, 321-328.
12. **Singh, H.P., Ruiz-Espejo, M., (2003).** On linear regression and ratio-product estimation of a finite population mean. *The Statistician* 52(1), 59-67. <http://dx.doi.org/10.1111/1467-9884.00341>
13. **Singh, H.P., Solanki, R.S., (2011a).** A general procedure for estimating the population parameter in the presence of random non-response. *Pakistan Journal of Statistics* 27(4), 427-465.
14. **Sisodia, B.V.S. and Dwivedi, V.K. (1981).** A modified ratio estimator using coefficient of an auxiliary variable, *J. Ind. Soc. Agri. Stat.* 33, 13-18.
15. **Tailor, R. (2002).** Some estimation problems based on auxiliary information in sample surveys. *Ph.D. Thesis submitted to Vikram University, Ujjain, M.P., India.*
16. **Tailor, R. Kumar, M and Tailor, R. (2006).** Estimation of finite population mean using power transformation. *International journal of Agricultural and Statistical Science*, 2, No. 2, pp. 341-345.
17. **Upadhyaya, L.N. and Singh, H.P. and Vos, J.W.E. (1985).** On the estimation of population means and ratio using supplementary information. *Statistica Neerl.*, 39, 3, 309-318.

